RADIAL EXPANSION OF CYLINDRICAL LAYERS OF VISCOUS AND RHEOLOGICALLY COMPLEX FLUIDS

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We theoretically analyze the dynamics of the expansion of cylindrical layers of Newtonian, power-law, and elastoviscous liquids, as well as an elastic body. We consider inertial expansion and expansion induced by gas pressure inside the cavity.

One of the basic elements of technological processes of obtaining products from polymer materials is inflation formation [1]. Inside cavities of a liquid polymer preparation an increased pressure is created, usually via the injection of air into the cavity, and the cavity begins to expand under the action of the internal pressure. From the practical point of view, as applied to inflation and also to other processes in which radial expansion of cylindrical liquid layers occurs, the interesting parameters are the rate of expansion, the thickness of the cavity wall, and the radius of the cavity. The rheological behavior of the liquid significantly affects these parameters. The purpose of the present paper is to study theoretically the effect of the rheological behavior of the liquid on the dynamics of the expansion of a liquid layer. The following values of the physical parameters are typical for liquid polymer materials [1-3]: $\rho \sim 10^3$ kg/m^3, $\mu \sim 10^2$-10^4 Pa-sec, $\alpha \sim 0.03$-0.05 J/m^2, $\theta \sim 10^{-2}$-10 sec, $G \sim 10$-10^3 Pa, $\alpha_1 = 0.1$. In many cases [1-4] the only rheological parameter different from a Newtonian fluid is the viscosity, and the liquid is characterized by a power law (for example, in [1, 4] $K \sim 10^2$-10^6 kg/(m$\cdot$sec^{2-n}), $n \simeq 0.5$).

Assuming that the liquid is incompressible during the radial expansion of the cylindrical layer, we study two regimes: inertial expansion (impulse) and inflation due to gas pressure on the inner walls of the cavity inside the layer. In the first case the kinetic energy of the layer of fluid per unit length E is specified. Then assuming axisymmetric flow and the incompressibility condition, the initial velocity of the outer boundary of the layer v_{20} is given by

$$v_{20} = \sqrt{\frac{E}{\pi\rho r_{20}^2 \ln(r_{20}/r_{10})}}. \tag{1}$$

As a scale of velocity we choose the quantity

$$v_0 = \sqrt{2E/[\pi\rho(r_{20}^2 - r_{10}^2)]}, \tag{2}$$

and with the help of (1) and (2) we obtain an expression for the dimensionless initial velocity of the outer boundary of the layer for the case of inertial expansion

$$v^0 = \sqrt{\frac{Q}{\ln[1/(1-Q)]}}, \quad Q = 1 - \frac{r_{10}^2}{r_{20}^2}. \tag{3}$$

We note that v_0 is the velocity of a layer of ideal fluid without surface tension in the limit $t \to \infty$.

In the other case, where the expansion of the layer of liquid is driven by gas pressure inside the cavity, the internal energy of the gas per unit length E_1 is given and the initial velocity of the liquid is naturally assumed to be zero. In this case as a scale of velocity we use the quantity v_{01} as given by (2), except that E is replaced by E_1. We note that v_{01}

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is the velocity which a layer of ideal fluid, without surface tension, acquires from inflation into a vacuum in the limit $t \to \infty$, when the energy of the gas is completely transformed into the kinetic energy of the fluid.

The initial gas pressure is obtained from the Clapeyron equation and is equal to

$$p_{10} = (\gamma - 1) \frac{E_1}{\pi r_{10}^2}. \tag{4}$$

Typically [1, 4] the pressure p_{10} is of order 10^5-10^7 Pa; this must be considered a lower bound to the possible range of p_{10}. The quantity r_{20} is of order 10^{-2} m, and the radius r_{10} can be either much smaller than r_{20} or comparable to it. The energy E_1 (for $p_{10} = 10^7$ Pa, $\gamma = 1.4$, $r_{10} = 0.4 \cdot 10^{-2}$ m) is, according to (4), about 10^3 J/m, and the velocity v_{01}, according to (2), for $r_{20} = 10^{-2}$ m and $\rho = 10^3$ kg/m^3, is of order 10^2 m/sec.

Assuming that the expansion of the gas is adiabatic and using (4), we can find the gas pressure inside the cavity at any time during the motion

$$p_1 = \frac{(\gamma - 1) Q (1 - Q)^{\gamma - 1}}{2 r_1^{2\gamma}}. \tag{5}$$

Here we have assumed that the gas pressure does not vary over the cross section of the cavity, which will be correct when the velocity of expansion is small compared to the speed of sound. The inner radius of the cavity is related to the outer radius by the following relation, which follows from the conservation of mass for the liquid: $r_1 = \sqrt{r_2^2 - Q}$.

The equation of continuity and the equation of motion of the liquid in the case of radial motion, written in terms of the dimensionless quantities, have the forms

$$\frac{\partial rv}{\partial r} = 0, \quad \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) - \frac{\tau_{\theta\theta}}{r}. \tag{6}$$

The equation of motion (6) can be integrated with the help of the boundary conditions for the radial stresses

$$(-p + \tau_{rr})|_{r=r_1} = -p_1 + \frac{1}{\mathrm{We}\, r_1}, \quad (-p + \tau_{rr})|_{r=r_2} = -p_2 - \frac{1}{\mathrm{We}\, r_2}. \tag{7}$$

With the help of the integral of the equation of continuity in (6) ($v = v_2 r_2 / r$), Eq. (5), and the boundary conditions (7), we integrate the equation of motion (6) between the inner and outer boundaries of the liquid layer and obtain

$$\frac{dv_2}{dt} = \left\{ \left[-\ln \frac{r_2}{\sqrt{r_2^2 - Q}} + \frac{Q}{2(r_2^2 - Q)} \right] v_2^2 + \frac{(\gamma - 1) Q (1 - Q)^{\gamma - 1}}{2 (r_2^2 - Q)^\gamma} - \right.$$
$$\left. - p_2 - \frac{1}{\mathrm{We}} \left(\frac{1}{r_2} + \frac{1}{\sqrt{r_2^2 - Q}} \right) + I \right\} \Big/ \left[r_2 \ln \frac{r_2}{\sqrt{r_2^2 - Q}} \right],$$
$$I = \int_{\sqrt{r_2^2 - Q}}^{r_2} \frac{\tau_{rr} - \tau_{\theta\theta}}{r} dr, \quad \frac{dr_2}{dt} = v_2. \tag{8}$$

For inertial expansion we have $v_2 = v^0$ and $r_2 = 1$ at $t = 0$; for inflation driven by gas pressure we have $v_2 = 0$ and $r_2 = 1$ at $t = 0$.

The deviator stresses τ_{rr} and $\tau_{\theta\theta}$ in (8) must be related to the expansion velocity of the layer and its radius with the help of the rheological equation of state; then (8) will be closed and we can calculate the expansion of the layer. For a Newtonian fluid $\tau = 2\mu D$, and for the radial motion considered here

$$\tau_{rr}-\tau_{\theta\theta}=-\frac{4}{\mathrm{Re}}\frac{r_2v_2}{r^2},\quad I=-\frac{2Q}{\mathrm{Re}}\frac{v_2}{r_2(r_2^2-Q)}. \tag{9}$$

For the typical values of the parameters $\rho = 10^3$ kg/m^3, v_{01} = 60 m/sec, $r_{20} = 10^{-2}$ m, μ = 10^2-10^4 Pa-sec, the Reynolds number Re varies between 0.06 and 6.

For a power-law liquid with the rheological equation of state [2] $\tau=2K[2\,\mathrm{Sp}\,\mathbf{D}^2]^{\frac{n-1}{2}}\mathbf{D}$ we have

$$\tau_{rr}-\tau_{\theta\theta}=-K_1\left(\frac{r_2v_2}{r^2}\right)^n;\quad I=-\frac{K_1}{2n}(r_2v_2)^n\left[\frac{1}{(r_2^2-Q)^n}-\frac{1}{r_2^{2n}}\right]. \tag{10}$$

Putting K = 10^2 kg/(m•sec$^{3/2}$), n = 0.5, $\rho = 10^3$ kg/m^3, v_{01} = 60 m/sec, $r_{20} = 10^{-2}$ m, the dimensionless complex K_1 is about 10^{-2}.

For an "elastic liquid" (i.e. a flexible elastic body) we have $\tau=2G(\mathbf{B}-\alpha_1\mathbf{B}^{-1})$ [3] and hence

$$\tau_{rr}-\tau_{\theta\theta}=2T\left(\frac{1-r_2^2+r^2}{r^2}-\frac{r^2}{1-r_2^2+r^2}\right),$$
$$I=T\left[\frac{Q(1-r_2^2)}{r_2^2(r_2^2-Q)}+\ln\frac{r_2^2}{r_2^2-Q}+\ln(1-Q)\right]. \tag{11}$$

For the typical values G = 10-10^3 Pa, α_1 = 0.1, $\rho = 10^3$ kg/m^3, v_{01} = 60 m/sec, we obtain T $\sim$ 10^{-6}-10^{-4}.

Before discussing the case of an elastoviscous liquid, we consider the aymptotic behavior of an initially thin liquid layer for which Q << 1. In this case (8) reduces to

$$\frac{dv_2}{dt}=\frac{\gamma-1}{r_2^{2\gamma-1}}-\frac{2r_2}{Q}p_2-\frac{4}{\mathrm{We}\,Q}+I_1,\quad \frac{dr_2}{dt}=v_2, \tag{12}$$

where the quantity I_1 is equal to the following expressions for a Newtonian, power law, and "elastic" liquid, respectively:

$$I_1=-\frac{4}{\mathrm{Re}}\frac{v_2}{r_2^2},\quad I_1=-\frac{K_1}{r_2}\left(\frac{v_2}{r_2}\right)^n,\quad I_1=-2T\left(r_2-\frac{1}{r_2^3}\right). \tag{13}$$

Note that for a thin layer r_2 and v_2 can be considered as the radius and velocity of the mid-surface of the film, and $Q = 2h_0/r_0$.

We consider now an elastoviscous Maxwellian liquid with the rheological relation [2]

$$\frac{\partial\tau}{\partial t_*}+(\mathbf{v}\cdot\nabla)\tau=-\frac{\tau}{\theta}+\tau\cdot\mathbf{D}+\mathbf{D}\cdot\tau+\tau\cdot\Omega-\Omega\cdot\tau+\frac{2\mu}{\theta}\mathbf{D}. \tag{14}$$

For the case of radial motion, and assuming the film is thin, we have at once (12) for the motion of the elastoviscous liquid, where

$$I_1=\frac{\tau_{rr}-\tau_{\theta\theta}}{r_2},\quad \frac{d\tau_{rr}}{dt}=\left(-\frac{1}{\theta_1}-2\frac{v_2}{r_2}\right)\tau_{rr}-2S\frac{v_2}{r_2},$$
$$\frac{d\tau_{\theta\theta}}{dt}=\left(-\frac{1}{\theta_1}+2\frac{v_2}{r_2}\right)\tau_{\theta\theta}+2S\frac{v_2}{r_2}. \tag{15}$$

When t = 0 we assume $\tau_{rr} = \tau_{\theta\theta} = 0,\ S = (\mathrm{Re}\,\theta_1)^{-1}$.

For velocities $v_{01} \sim 1\text{-}10^2$ m/sec, $\theta \sim 10^{-2}\text{-}10$ sec, $r_0 \sim 10^{-2}\text{-}10^{-1}$ m, the dimensionless relaxation time θ_1 varies between 10^{-1} and 10^5.

Equation (12) and the closing relations (13) or (15) can also be obtained directly from the quasi-two-dimensional dynamical equations of a thin film [5, 6] by averaging all quantities over the cross section of the film.

The simplified equation (12) can be solved analytically in several important cases. We consider first the inertial expansion of the liquid layer into a vacuum. In this case we put $\gamma = 1$ and $p_2 = 0$ in (12) and the integration is carried out with the conditions $r_2 = v_2 = 1$ at t = 0 (here v_0 is the initial velocity of expansion). For a layer of ideal fluid (Re = ∞) with surface tension, we obtain from integrating (12)

$$r_2 = 1 + t - \frac{2t^2}{Q\,\mathrm{We}}. \tag{16}$$

After a time t = QWe/4 the cylindrical film has expanded to its maximum size of $r_2 = 1 + \mathrm{QWe}/8$ and then begins to collapse. For $v_0 \sim 10\text{–}60$ m/sec, $\rho = 10^3$ kg/m^3, $r_0 = 10^{-2}$ m, $\alpha \sim 0.03 - 0.05$ J/m^2, we have We $\sim 10^4\text{-}10^6$. Putting Q = 10^{-1} and We = 10^4, we obtain a maximum radius of the film of order 1 m.

In the case of a viscous Newtonian fluid, ignoring surface tension, we obtain from (12) that when Re > 4 the cylindrical layer expands to infinity and the dependence of the radius on time is given by

$$\frac{r_2 - 1}{1 - 4/\mathrm{Re}} - \frac{4/\mathrm{Re}}{(1 - 4/\mathrm{Re})^2} \ln\left[\frac{4}{\mathrm{Re}} + \left(1 - \frac{4}{\mathrm{Re}}\right) r_2\right] = t. \tag{17}$$

It follows from (17) that when Re > 4 the viscous layer, expanding inertially, approaches infinity with a finite velocity $v_{2\infty} = (\mathrm{Re} - 4)/\mathrm{Re}$. This is because when Re > 4 the kinetic energy of the layer at the initial instant of time exceeds the total energy dissipated during the expansion to infinity; the ratio of the kinetic energy to the energy dissipated during the expansion to infinity is equal to Re/(8 − 16/Re) and exceeds unity for Re > 4. Only when Re = 4 does this ratio become equal to one and then

$$r_2 = \sqrt{1 + 2t}, \quad v_{2\infty} \to 0. \tag{18}$$

When Re < 4 the initial kinetic energy is dissipated during the expansion of the layer to a finite radius given by $r_{2\max} = 4/(4-\mathrm{Re})$. The dependence of the radius on time is given by (17) as before, which shows that in general the value $r_{2\max}$ is reached after an infinite time.

We note that for v_0 = 60 m/sec and Re = 5 the layer approaches infinity with a velocity of 12 m/sec; when $r_0 = 10^{-2}$ m and Re = 3.5 the layer expands to a maximum radius of $8 \cdot 10^{-2}$ m.

Similar behavior occurs for the inertial expansion of a power-law liquid layer (surface tension is again neglected). Integrating (12) and (13) we find that when $K_1 \leq n/(2-n)$ the cylindrical film expands to infinity and $v_{2\infty} = [1 - (2-n)K_1/n]^{1/(2-n)}$; for $K_1 = 10^{-2}$, n = 0.5, v_0 = 60 m/sec we obtain a finite velocity for the approach to infinity of 58.8 m/sec. When $K_1 > n/(2-n)$ the layer expands to a finite maximum radius $r_{2\max} = [1 - n/[(2-n)\cdot K_1]]^{-1/n}$, and gradually slows down as it approaches this radius.

In the case of inertial expansion of a thin layer of "elastic liquid" we find, integrating (12) and (13), that the layer expands to a finite radius $r_{2\max} = (2\sqrt{2T})^{-1} + \sqrt{1 + 1/(8T)}$, then the layer begins to contract until its radius reaches the value $r_{2\min} = -(2\sqrt{2T})^{-1} + \sqrt{1 + 1/(8T)}$. Since energy losses are zero for an "elastic liquid" this process will repeat periodically. For T = 10^{-4}, $r_0 = 10^{-2}$ m the layer expands to a maximum radius of 0.7 m.

The results for the inertial expansion of a thin layer of elastoviscous liquid were obtained by a numerical integration of (12) and (15) and are shown in Figs. 1 and 2.

As in the case of a Newtonian fluid, when Re $\gg 4$ the film approaches infinity with a certain velocity $v_{2\infty}$. When Re 4 the radius of the film approaches a finite value $r_{2\infty}$ in the limit $t \to \infty$. When $\theta_1 \leqslant 4$ the dependence of $r_{2\infty}$ and $v_{2\infty}$ on Re can be described with satisfactory accuracy (the error is $\sim 5\%$) with the help of the relations between $v_{2\infty}$, r_{2max} and Re given above for a Newtonian fluid. When $\theta_1 > 4$ the elastic forces begin to affect the quantities $v_{2\infty}$ and $r_{2\infty}$. For example when Re = 3.75 we have $r_{2max} = 16$ for a Newtonian fluid, whereas for an elastoviscous liquid with $\theta_1 = 10$ we have $r_{2\infty} = 5.57$. Assuming $\theta = 10^{-2}$ sec, $v_0 = 10$ m/sec, $r_0 = 10^{-2}$ m ($\theta_1 = 10$), we find that a film of elastoviscous liquid with Re = 3.75 ($\mu = 0.266 \cdot 10^2$ Pa-sec) expands to a maximum radius of $5.57 \cdot 10^{-2}$ m, whereas a film of Newtonian fluid with the same Reynolds number expands to the value 0.16 m.

For large θ_1 or small Re the vibrations of the film are damped. The relaxation time is about a quarter of a period of vibration. The internal stresses increase from zero at the initial instant of time up to a maximum value after a time approximately equal to the relaxation time of the material.

The simplified equations for a thin layer can be used to analyze the effect of the internal gas pressure in the cavity and the back pressure p_2 of the surrounding medium on the dynamics of the expansion of the layer. For the case of an ideal fluid we find from (12)

$$\frac{dr_2}{dt} = v_2 = \pm \sqrt{-\frac{1}{r_2^{2(\gamma-1)}} + 1 + \frac{8}{Q\,\mathrm{We}}(1 - r_2) + \frac{2p_2}{Q}(1 - r_2^2)}. \tag{19}$$

Obviously we must have $p_2 \leqslant (\gamma - 1)Q/2 = p_1|_{t=0}$.

In the absence of back pressure and surface tension ($p_2 = 0$, We $= \infty$) the film expands to infinity under the action of the gas pressure inside the cavity and the velocity approaches its largest possible value $v_{2max} = 1$. When there are surface tension forces or back pressure of the surrounding medium the radius of the film oscillates. A numerical solution of (19) shows that when We $= \infty$, $p_2 = 0.00333$, $Q = 0.1$, $\gamma = 1.4$ the film oscillates between finite limits $1 \leqslant r_2 \leqslant 3.17$. The maximum value of the velocity during these oscillations is determined by (19) and given by (We $= \infty$)

$$v_{2max} = \sqrt{1 + \frac{2p_2}{Q}\left\{1 - \frac{\gamma}{\gamma - 1}\left[\frac{(\gamma - 1)Q}{2p_2}\right]^{1/\gamma}\right\}}. \tag{20}$$

For the above values of the parameters, $v_{2max} = 0.48$ and is reached when $r = 1.9$. Using the velocity scale $v_{01} = 60$ m/sec ($E_1 \sim 10^3$ J/m) and $\rho = 10^3$ kg/m^3, for the above values of Q and γ we obtain a pressure inside the cavity at the initial time equal to $0.72 \cdot 10^5$ Pa and a back pressure $0.12 \cdot 10^5$ Pa. At the initial time, when the radius of the film is, for example, 10^{-2} m, its velocity is zero and when the film radius is $1.9 \cdot 10^{-2}$ m the velocity reaches its maximum value of 28.8 m/sec. The maximum radius of the film during the oscillations is $3.17 \cdot 10^{-2}$ m.

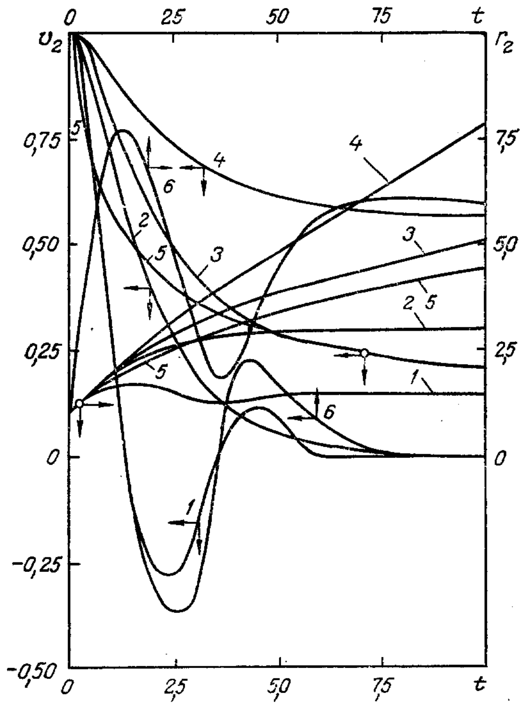
If the back pressure is small in comparison with the initial pressure of the gas inside the cavity, i.e. $p_2 \ll (\gamma - 1)Q/2$, then (20) can be simplified and reduces to

$$v_{2max} = 1 - \frac{\gamma}{2}\left[\frac{2p_2}{(\gamma - 1)Q}\right]^{\frac{\gamma-1}{\gamma}}. \tag{21}$$

Viscous forces can further decrease the value v_{2max}. It is also obvious that in the presence of internal pressure (and $p_2 = 0$, We $= \infty$) the viscous forces, unlike the case of inertial expansion, can never stop the film at a finite distance; it always expands to infinity (this follows from (12) and (13)).

We consider the maximum velocity and radius for the expansion of a film of "elastic liquid" in the presence of back pressure. We find from (12)

$$v_2 = \pm \sqrt{1 - \frac{1}{r_2^{2(\gamma-1)}} + \frac{8}{Q\,\mathrm{We}}(1 - r_2) + 2\left(\frac{p_2}{Q} + T\right)(1 - r_2^2) + 2T\left(1 - \frac{1}{r_2^2}\right)}. \tag{22}$$



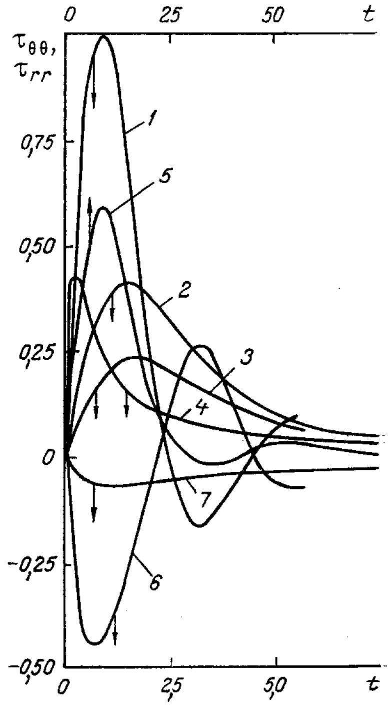
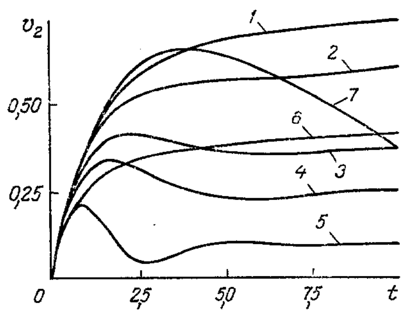


Fig. 1

Fig. 2

Fig. 1. Time dependence of the velocity and radius of a thin layer (Q = 0.02) of elastoviscous fluid for inertial expansion into a vacuum. θ_1 = 1: curve 1) Re = 0.9375; 2) Re = 2.5; 3) Re = 3.75; 4) Re = 7.5. For curves 5 and 6) Re = 3.75: 5) θ_1 = 0.1; 6) θ_1 = 10.

Fig. 2. Dependence of the azimuthal $\tau_{\theta\theta}$ (curves 1-5) and radial τ_{rr} (curves 6 and 7) stresses on time for inertial expansion into a vacuum of a thin layer (Q = 0.02) of elastoviscous fluid. θ_1 = 1.0: curves 1 and 6) Re = 0.9375; 2) Re = 3.75; 3 and 7) Re = 7.5. For curves 4 and 5) Re = 3.75: 4) θ_1 = 0.1; 5) θ_1 = 10.



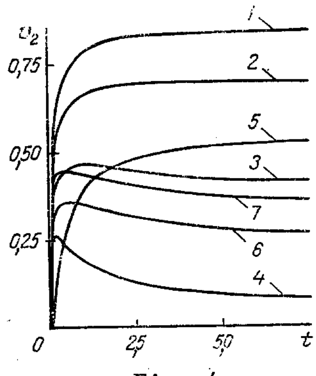


Fig. 3

Fig. 4

Fig. 3. Time dependence of the velocity of expansion into a vacuum (p_2 = 0) of a thin layer (Q = 0.02) of an elastoviscous fluid driven by pressure inside the cavity. θ_1 = 1.0: curve 1) Re = 15; 2) Re = 7.5; 3) Re = 3.75; 4) Re = 2.5; 5) Re = 0.9375. For curves 6 and 7) Re = 3.75: 6) θ_1 = 0.1; 7) θ_1 = 10.

Fig. 4. Time dependence of the velocity of expansion into a vacuum (p_2 = 0) of a layer of Newtonian fluid with Q = O(1) driven by pressure inside the cavity. γ = 1,4, Q = 0.9996: curve 1) Re = 60; 2) Re = 30; 3) Re = 15; 4) Re = 7.5. γ = 1.4, Re = 7.5: 5) Q = 0.75; 6) Q = 0.99. 7) γ = 3.0, Re = 15, Q = 0.9996.

Assuming that $2(p_2/Q + T)/(\gamma - 1) << 1$ and neglecting the contribution of surface tension we obtain

$$v_{2\max} \simeq 1 - \frac{\gamma}{2}\left[\frac{2(p_2/Q+T)}{\gamma-1}\right]^{\frac{\gamma-1}{\gamma}}, \quad r_{2\max} \simeq \frac{1}{\sqrt{2(p_2/Q+T)}}. \tag{23}$$

The maximum velocity of the film is acquired at a distance $r_* \simeq \{(\gamma-1)/[2(p_2/Q+T)]\}^{1/2\gamma}$ (the latter three equations are valid when $r_* >> 1$). For $p_2 = 0$, $\gamma = 1.4$, $T = 10^{-4}$ we find $v_{2\max} = 0.92$, $r_{2\max} = 71$, $r_* = 15$. Hence in the case $v_{01} = 60$ m/sec, $r_0 = 10^{-2}$ m, the maximum velocity is 55 m/sec and is reached at a radius of 0.15 m; elastic forces in the liquid prevent the film from expanding to a radius larger than 0.71 m.

The results of a calculation of the radial expansion of a thin layer (Q = 0.02) of a viscoelastic fluid in the presence of pressure inside the cavity are shown in Fig. 3. We see that the characteristics of the expansion of the layer ($v_{2\max}$ and $v_{2\infty}$) depend significantly on the rheological parameters of the material. For example, when $\theta_1 = 10$ and Re = 3.75 we find for $v_{01} = 10$ m/sec the value $v_{2\max} \approx 6.5$ m/sec.

In the calculation of the motion of a layer of liquid with a large initial thickness (Q of order unity) a numerical integration of (8) must be done with the closing rheological equations (9) through (11) for a Newtonian, power-law, and "elastic" liquid, respectively. The calculations show that the qualitative nature of the phenomena observed in the analysis of the dynamics of a thin layer does not change when Q = O(1). For example, calculation of the radial inertial expansion of a Newtonian fluid with Q = 0.9996 shows that when Re = 7.5 the radius of the layer approaches a finite value $r_{2\max} = 1.3$, whereas when Re = 11.25 the layer expands to infinity.

In Fig. 4 we show the time dependence of the velocity of radial expansion for a Newtonian fluid with Q = O(1) in the presence of internal pressure (and $p_2 = 0$, We = ∞) for different values of Re and γ. We see that the layer accelerates to a value $v_{2\max}$ and then its velocity starts to decrease. The quantity $v_{2\max}$ and the time during which the layer accelerates decrease with increasing Q and γ and with decreasing Re. The calculations for large times show that the velocity decreases to a certain minimum value $v_{2\min}$ and then v_2 starts to slowly increase. For example when Q = 0.9996, γ = 1.4, Re = 3.75 the fluid layer accelerates to $v_{2\max} = 0.16$ during a time $t_1 = 0.1$ and then the velocity decreases to the value $v_{2\min} = 0.021$ which is reached at the time $t_2 = 50.1$. The velocity then slowly increases and finally ($t_3 = 200$) reaches the value 0.023. When $v_{01} = 60$ m/sec, $r_{02} = 10^{-2}$ m we obtain $t_1 = 0.167 \cdot 10^{-4}$ sec, $t_2 = 0.835 \cdot 10^{-2}$ sec, $v_{2\max} = 9.6$ m/sec.

NOTATION

E, kinetic energy of the liquid layer per unit length at the initial instant of time (in this case, i.e., inertial expansion, the characteristic velocity $V = v_0$); E_1, internal energy of the gas in the cavity at the initial time (in this case, i.e., expansion driven by the internal gas pressure, $V = v_{01}$); ρ, μ, α, density, viscosity, and surface tension of the liquid; r_1 and r_2, inner and outer radii of the layer in units of r_{20} (r_{10} and r_{20} are the initial values); p_{10}, initial pressure of the gas inside the cavity; v_{20}, initial velocity of motion of the outer boundary of the layer (dimensionless for inertial expansion: $v^\circ = v_{20}/v_0$); p_1, gas pressure inside the cavity, in units of ρv_0^2; γ, adiabatic index; r, radical distance, in units of r_{20}; v, velocity of the liquid, in units of V; p, pressure of the liquid, in units of ρV^2; τ_{rr} and $\tau_{\theta\theta}$, radial and azimuthal deviator stresses in the liquid, in units of ρV^2; t, time, in units of $r_{20}/V(t = t_*V/r_{20})$; p_2, pressure of gas external to the layer, in units of ρV^2; K and n, rheological parameters of a power-law liquid; α_1, dimensionless rheological parameter of an "elastic liquid"; G, modulus of elasticity; h_0 and r_0, initial thickness and radius of a thin layer (in the case of a thin film the radius r is in units of r_0); θ, relaxation time ($\theta_1 = \theta V/r_{20}$); τ and D, deviators of the stress tensor and deformation rate tensor; B, Green's tensor; Ω, rotation tensor; **v**, velocity vector of the fluid; Q, dimensionless parameter characterizing the initial thickness of the liquid layer; $We = \rho V^2 r_{20}/\alpha$, Weber number; $K_1 = 4K(2V/r_{20})^{n-1}/(\rho V r_{20})$; $T = G(1 + \alpha_1)/(\rho V^2)$; $S = \mu/(\theta\rho V^2)$, $Re = \rho V r_{20}/\mu$, Reynolds number.

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SETTLING TIME OF LIQUID IN TANKS UNDER THE ACTION OF MINOR OVERLOADING

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A theoretical model is proposed for determining the time t_0, which is the basis for the concept of ascent and division of large gas bubbles in liquid. The estimate of t_0 given by this model is in qualitative agreement with experiment.

One method of ensuring continuity of a liquid flow pumped out from a tank under reduced gravitation is to apply a small acceleration g to the tank ($g < g_0$). Under the action of this acceleration, the liquid flows toward the intake unit (conventionally, downward) and the bubbles of pressurization gas and liquid vapor which it contains move to the opposite wall of the tank (float upward).

To estimate the time of this process t_0, various models are proposed. In [1], for example, it was recommended that liquid motion be considered by analogy with the free fall of a solid in a field $g \ll g_0$ and that the fall time t_{fall} be determined from the formula

$$S = \frac{g t_{fall}^2}{2}. \tag{1}$$

Experiments [1] show that $t_0 > t_{fall}$, and it was recommended in [1] that the value $t_0 = (2\text{-}5) t_{fall}$ be taken. This means that in t_0, as well as t_{fall}, account is taken of the time of partial damping of the liquid, the time of bubble ascent, and possibly the duration of other processes.

The practical recommendation of [1] as regards determining t_0 is now justified, by considering the ascent of gas bubbles in the liquid under the action a small acceleration $g \simeq (10^{-2})\text{-}(10^{-4}) g_0$ rather than the fall of the liquid to the intake unit. It is assumed that the pressurization gas is initially concentrated in a single bubble, which is at the intake unit and then moves to the opposite end of the tank. As the bubble rises, it breaks down into several smaller bubbles, which also, in turn, break down further, thus creating a cluster of bubbles.

To describe the motion of an individual bubble, the semiempirical approach of [2] is used. According to [2], the velocity of steady motion of so-called large bubbles does not depend on their size

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